UDC 551.509.313

On the Efficient Reduction of Truncation Error in Numerical Weather Prediction Models

JOSEPH P. GERRITY, JR., RONALD D. McPHERSON, and PAUL D. POLGER
National Meteorological Center, National Weather Service, NOAA, Suitland, Md.

ABSTRACT—Second- and fourth-order accurate finite-difference approximations of the equations governing a free surface autobarotropic fluid are compared with each other and with a second-order approximation on a one-half mesh. It is concluded that once the mesh size has been reduced sufficiently to adequately resolve the scales of interest then further reduction in mesh size would be inefficient in comparison with the use of more accurate finite-difference approximations.

1. INTRODUCTION

The use of numerical integration methods to study the behavior of theoretical models of the atmosphere or to predict the evolution of an actual state of the atmosphere is subject to several limitations. One of these limitations involves the accuracy of the numerical integration methods employed in the calculations. In recent papers (Miyakoda et al. 1971, Wellck et al. 1971), the influence of the size of the horizontal gridpoint separation upon the accuracy of numerical solutions of complex baroclinic models was reported. The variation of gridpoint separation most directly influences the truncation error in the calculation of derivatives, but its also plays a role in the inner workings of the nonlinear and diabatic processes incorporated in the models (Manabe et al. 1970). Experience with a fine-mesh version of the National Meteorological Center primitive-equation baroclinic model indicates that the increased resolution also results in more accurate short-range (24-hr) forecasts.

The approach adopted in the work noted above involved the variation of the grid resolution alone; the finite-difference approximations were otherwise unaltered. Several papers (Miyakoda 1960, Crowley 1968, Grammeltvedt 1969) have been published that indicate that truncation error may be effectively reduced by the use of more accurate (i.e., higher order) finite-difference approximations. We are unaware of any previous work in which comparative integrations of the primitive meteorological equations have been conducted using fine-mesh and higher order difference approximations. The present paper is intended as a modest contribution in this regard.

We will show that, at least with respect to the primitive, free-surface barotropic model, the use of higher order finite-difference approximations offers an efficient alternative to the reduction of the gridpoint separation as a method for the reduction of truncation error. The efficiency factor arises from the fact that computation requirements vary as the inverse third power of the grid separation. The halving of gridpoint spacing requires an eightfold increase in computing effort to attain a forecast valid at a fixed time after the initial moment. In contrast, the use of a higher order scheme requires no increase in the number of gridpoints. The required calculations at each of the points are only increased by a small amount, and the time step admitted by the linear stability constraint is reduced by some 30 percent as compared with 50 percent in the half-mesh case.

2. FORMULATION OF A HIGHER ORDER APPROXIMATION

The order of a finite-difference approximation is defined as the exponential power to which the grid interval, Δx, is raised in the leading term of the residual. For example, a function, $f(x)$, which is analytic, possesses in some neighborhood of a point x_j the power series representation

$$f(x_j+\Delta x)=f(x_j)+\frac{\partial f}{\partial x}\Delta x+\frac{\partial^2 f}{\partial x^2}\frac{\Delta x^2}{2!}+O(\Delta x^3) \tag{1}$$

where the symbol $O(\Delta x^3)$ means that we omit residual terms that are (small) of the order Δx^3. By combining eq (1) with a similar expression for $f(x_j-\Delta x)$, one may derive the result

$$f(x_j+\Delta x)-f(x_j-\Delta x)=2\frac{\partial f}{\partial x}\Delta x+O(\Delta x^3). \tag{2}$$

A division of eq (2) by $2\Delta x$ yields the second-order, accurate, centered, finite-difference approximation for the first derivative,

$$\frac{\partial f}{\partial x}=\frac{f(x_j+\Delta x)-f(x_j-\Delta x)}{2\Delta x}+O(\Delta x^2). \tag{3}$$

A common misinterpretation of eq (3) should be put to rest. Some ask the question, "If Δx is a large number (e.g., 381 km), why does not a higher power of Δx in the residual imply the existence of a greater error in the approximation?" To answer this objection, one may refer back to eq (1) and note that the next term in the power series would have been

$$\frac{\partial^3 f}{\partial x^3}\frac{(\Delta x)^3}{3!}. \tag{4}$$

The magnitude of this term depends upon the third derivative of the function, $f(x)$, as well as the power of Δx. Let us suppose that $f(x)$ is the trigonometric function

$$f(x)=\sin\frac{2\pi x}{L} \tag{5}$$

where L is the wavelength or scale of the trigonometric function.

When eq (5) is used in eq (4), one gets

$$\frac{(2\pi)^3}{3!}\left(\frac{\Delta x}{L}\right)^3 \sin\left(\frac{2\pi x}{L}\right). \tag{6}$$

It is, therefore, clear that an increase in the order of an approximation will lead to increased accuracy only if the grid spacing, Δx, is small compared with the scale of the function being differentiated. It is imperative that the grid resolution be sufficiently refined with respect to the scale of the fields that one wishes to calculate. As a general rule (Miyakoda 1960, Grammeltvedt 1969), one should require $(\Delta x/L)$ to be about 0.1.

In developing a higher order scheme for approximating the terms that appear in the equations of meteorological dynamics, we have followed the approach used by Shuman (1962). The derivative is first estimated midway between gridpoints by a centered-difference formula. The derivative is then transferred to gridpoints by an interpolation. Two symbolic operators were defined by Shuman,

$$f_x=\frac{1}{\Delta x}\,(f_{j+1/2}-f_{j-1/2}) \tag{7}$$

and

$$\overline{f}^x={}_{1/2}(f_{j+1/2}+f_{j-1/2}). \tag{8}$$

The first symbol [eq (7)] denotes an approximation to the first derivative of the function; the second symbol [eq (8)] denotes a simple interpolation. Since the gridpoints exist only for integer values of j, and the discrete function, f_j, is defined only at the gridpoints, eq (7) and (8) are applicable individually only midway between neighboring gridpoints. But, when symbols are combined; that is, when the derivative is averaged, the result is defined at a gridpoint. The second-order accurate approximation to the first derivative given in eq (3) is symbolically denoted by $\overline{f}^x_x$.

By analogy, we approached the approximation of the first derivative in two steps. We first determined the centered, fourth-order approximation for the derivative valid midway between gridpoints; that is,

$$f_{x_h}=\frac{9}{8}\left(\frac{f_{j+1/2}-f_{j-1/2}}{\Delta x}\right)-\frac{1}{8}\left(\frac{f_{j+3/2}-f_{j-3/2}}{3\Delta x}\right). \tag{9}$$

The subscript, h, is used to denote the higher order nature of the approximation.

The second step is the development of a formula for interpolating the derivative back to a gridpoint. A variety of possibilities exist for the interpolation. In this paper, we report on only one that was developed through symmetry considerations and the imposition of constraints on the accuracy of the low wave number interpolation. Denoting this averaging operator by an overbar x_h, we write

$$\overline{f}^{x_h}=\frac{9}{8}\left(\frac{f_{j+1/2}+f_{j-1/2}}{2}\right)-\frac{1}{8}\left(\frac{f_{j+3/2}+f_{j-3/2}}{2}\right). \tag{10}$$

Once more, the final approximation for the first derivative may be expressed by a combination of the two operators. One obtains

$$\overline{f}^{x_h}_{x_h}=\frac{1}{2\Delta x}\left[\frac{87}{64}\,(f_{j+1}-f_{j-1})-\frac{12}{64}\,(f_{j+2}-f_{j-2})+\frac{1}{192}\,(f_{j+3}-f_{j-3})\right]. \tag{11}$$

A comparison of this formula with the fourth-order or "five point" scheme given by Miyakoda (1960) suggests their near equivalence. It is likely that Crowley's (1968) experience would be replicated if the last term were suppressed. However, the last term in eq (11) does add to the accuracy of the approximation for functions with large scales, and it is largely responsible for the theoretical superiority of the higher order scheme to the use of a fine mesh in the low wave number portion of the spectrum.

The truncation error of the fourth-order approximation [eq (11)] may be compared with that of the second-order approximation [eq (3)] and with the error associated with the second-order approximation when a finer (one-half) mesh size is used. To do this comparison, we consider a complex trigonometric function, e^{ikx}, and examine the ratio of the finite difference first derivative to the analytic first derivative, as the wave number, k, is varied.

Since we are dealing with discrete functions, the wave number is not uniquely determined because of aliasing. We shall proceed on the premise that the wave number to be associated with a given set of gridpoint values is that lying within the interval

$$0\leq k<\frac{\pi}{\Delta x}. \tag{12}$$

For concreteness, we assume a fundamental interval covered by 30 grid intervals. The allowable values of k on such a domain are related to the Fourier integer indices, m, by

$$k=\frac{2\pi m}{30\Delta x}$$

with m running between 0 and 15, the latter value being associated with the limiting two-grid interval wave on the regular (i.e., nonfine mesh) grid.

In table 1, we have tabulated the ratio of the finite-difference approximation of the first derivative to the

TABLE 1.—*The ratio of finite-difference approximation of the first derivative to the analytic value for second-order accurate schemes on a regular and one-half mesh and for a fourth-order accurate scheme as a function of Fourier harmonic index,* m, *on a fundamental interval containing 30 gridpoint intervals*

m	Second order	Fourth order	Second-order half mesh
1	0. 993	1. 000	0. 998
2	. 971	0. 999	. 993
3	. 936	. 996	. 984
4	. 887	. 987	. 971
5	. 827	. 969	. 955
6	. 757	. 939	. 936
7	. 678	. 892	. 913
8	. 594	. 827	. 887
9	. 505	. 743	. 858
10	. 414	. 640	. 827
11	. 323	. 521	. 793
12	. 234	. 391	. 757
13	. 149	. 256	. 718
14	. 071	. 123	. 678
15	. 000	. 000	. 637

analytic value as a function of the Fourier integer index, m, for the three alternate difference schemes.

In anticipation of the results of a trial numerical integration reported later in this paper, we note that the fine-mesh and fourth-order schemes have equivalent accuracy at the sixth harmonic and that both of these schemes are considerably more accurate than is the second-order scheme. It is of some interest to note the slightly superior accuracy of the fourth-order scheme for the low values of m. In our view, only if the fourth-order scheme is more efficient than the fine-mesh scheme would it be legitimate to give preference to the fourth-order scheme's improved accuracy over the fine-mesh scheme.

3. TRUNCATION ERROR IN NONLINEAR TERMS

Grammeltvedt (1969) and Lilly (1965) have discussed the nonlinear stability properties of various finite-difference schemes for the equations of meteorological dynamics. In broad agreement with their results and our own experience (Robert et al. 1970), the problem of nonlinear stability control seems to lie in the suppression of the development and interaction of significantly different numerical solutions on alternate Richardson lattices (Platzman 1958, 1963, Richardson 1965) of the finite-difference equations. In the works of Lilly, Miyakoda, and Crowley, previously cited one finds clever devices used to uncover some facets of the nonlinear problem but rigorous analysis is not really possible. In this paper, we have confined ourselves to a simple inspection of the response given by the second-order and fourth-order approximation of an advective term.

The term $u(\partial\theta/\partial x)$ is typical of the nonlinear terms encountered in meteorological dynamics. We presume that the variables u and θ are at some time describable by Fourier components

$$u(x)=Ue^{irx}$$
and
$$\theta(x)=Te^{isx}. \tag{13}$$

The analytic value of the term is, therefore,

$$u\frac{\partial\theta}{\partial x}=isu\theta. \tag{14}$$

We then consider the two finite-difference approximations to the term

$$\overline{\bar{u}^x\theta_x}^x$$
and
$$\overline{\bar{u}^x\theta_{x_h}}^{x_h} \tag{15}$$

in which the symbols are as defined earlier. Note that the advecting wind, u, has been averaged using the same operator in both expressions. This is done here to be compatible with the scheme used in the two-dimensional integration reported later. Using the discrete form of the functions in eq (13); that is,

$$u_j=Ue^{ijr\Delta x}$$
and
$$\theta_j=Te^{ijs\Delta x} \tag{16}$$
with
$$0<r\Delta x<\pi$$
and
$$0<s\Delta x<\pi, \tag{17}$$

the ratio of the finite-difference approximation [eq (15)] to the analytic value [eq (14)] of the nonlinear term was calculated as a function of $r\Delta x/\pi$ and $s\Delta x/\pi$. The result of this calculation is tabulated in table 2.

An inspection of these results certainly is indicative of the difficulty of achieving a modicum of accuracy in a nonlinear integration by finite-difference methods. It will be noted that half of the table contains negative responses. These values occur for all of the interactions that involve an aliased result. Only in the lower left corner of the table does one encounter values that seem modestly good. The second-order scheme has an accuracy greater than 90 percent only when both the advecting wind and the advected quantity are resolved by more than 20 gridpoints. In the fourth-order scheme, this resolution is relaxed to just 10 gridpoints. One may estimate that a fourth-order scheme yields nonlinear term estimations approximately equivalent to those attainable with a second-order scheme on a half mesh.

The slightly larger, erroneous responses given by the fourth-order scheme in the aliasing domain was previously noted by Grammeltvedt (1969). We have not seriously pursued this question because of its complexity and the likelihood that nonlinear instability may be controlled (avoided) by techniques not directly related to the accuracy of the finite-difference schemes.

TABLE 2.—*Ratio of finite-difference to analytic value of an advection term as a function of wave numbers s and r of the advected quantity and the advecting wind, respectively. The upper number is for the second-order scheme and the lower number is for the fourth-order scheme.*

$s\Delta x/\pi$ \ $r\Delta x/\pi$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1.0	−0.10	−0.19	−0.26	−0.30	−0.32	−0.30	−0.26	−0.19	−0.10
	−.17	−.31	−.42	−.47	−.46	−.41	−.33	−.23	−.12
0.9	.00	−.10	−.19	−.26	−.29	−.29	−.26	−.19	−.10
	.00	−.18	−.32	−.42	−.45	−.42	−.35	−.25	−.13
.8	.12	.00	−.11	−.19	−.24	−.26	−.24	−.19	−.11
	.20	.00	−.18	−.32	−.39	−.40	−.35	−.26	−.13
.7	.25	.12	.00	−.10	−.18	−.22	−.22	−.18	−.10
	.41	.20	.00	−.17	−.29	−.34	−.33	−.25	−.14
.6	.38	.25	.12	.00	−.10	−.16	−.18	−.16	−.10
	.60	.41	.20	.00	−.16	−.25	−.27	−.23	−.13
.5	.52	.39	.25	.11	.00	−.08	−.13	−.13	−.08
	.75	.59	.39	.18	.00	−.13	−.20	−.19	−.12
.4	.65	.52	.38	.23	.10	.00	−.07	−.09	−.07
	.86	.73	.56	.36	.16	.00	−.10	−.14	−.10
.3	.77	.65	.51	.35	.21	.09	.00	−.05	−.05
	.93	.84	.69	.51	.32	.14	.00	−.07	−.07
.2	.87	.76	.62	.47	.32	.18	.07	.00	−.02
	.97	.90	.79	.63	.45	.26	.11	.00	−.04
.1	.94	.84	.72	.57	.41	.27	.14	.05	.00
	.98	.94	.84	.71	.55	.37	.20	.07	.00

4. LINEAR COMPUTATIONAL STABILITY

The question arises, "Is the linear stability criterion modified when one uses a higher order finite difference approximation for the spatial derivatives?" The answer appears to be yes. The following simple analysis is offered in demonstration.

Using the leapfrog scheme for the time derivative, one may consider the simple equation

$$\frac{\partial f}{\partial t}=-c\,\frac{\partial f}{\partial x},\tag{18}$$

in which c is a constant. The spatial derivative will be approximated by both the second- and fourth-order schemes,

$$\bar{f}_t^t=-c\bar{f}_x^x\tag{19}$$

and

$$\bar{f}_t^t=-c\bar{f}_{x_h}^{x_h}.\tag{20}$$

The solution is sought in the form

$$f(j\Delta x,\,n\Delta t)=\zeta^n e^{ikj\Delta x}.\tag{21}$$

The requirement for computational stability is that $|\zeta|<1$, and this leads to the constraint

$$\Delta t\leq\frac{1}{\hat{k}c}\tag{22}$$

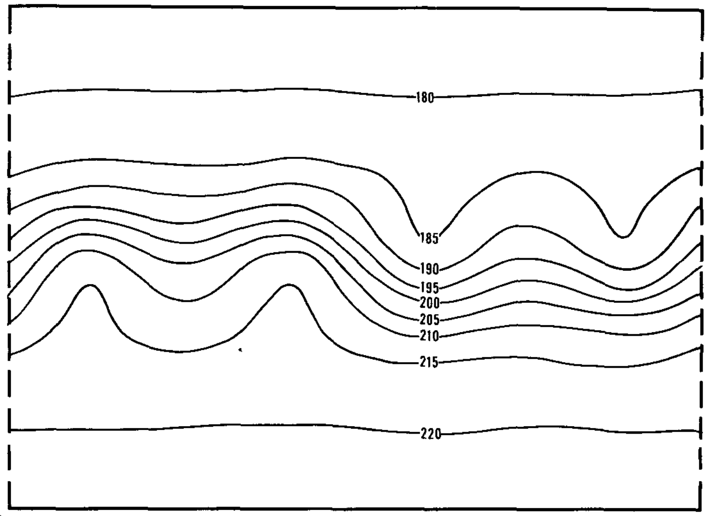

FIGURE 1.—The initial distribution of geopotential height depicted by isopleths drawn at 50-m intervals. The channel walls are shown as solid horizontal lines, the cyclic boundaries as dashed vertical lines. The domain is covered by 31 gridpoints in the horizontal (east–west) and 23 gridpoints in the vertical (north–south).

with

$$\hat{k}=k\,\frac{\sin k\Delta x}{k\Delta x}\tag{23}$$

for eq (19), and

$$\hat{k}=\frac{k}{64}\left(87\,\frac{\sin k\Delta x}{k\Delta x}+\frac{\sin 3k\Delta x}{3k\Delta x}-24\,\frac{\sin 2k\Delta x}{2k\Delta x}\right)\tag{24}$$

in the case of eq (20).

The limit on Δt is associated with the largest value of $\hat{k}$. If $(\Delta t)_2$ is the limit associated with the second-order scheme, then by comparison of eq (23) and (24), one sees that the limit on Δt for the fourth-order scheme $(\Delta t)_4$ is approximately

$$(\Delta t)_4=\frac{64}{87}\,(\Delta t)_2=0.735(\Delta t)_2.\tag{25}$$

One finds, in this way, that the use of the fourth-order scheme requires a 30-percent reduction in the time step used in the integration. This is not an excessive penalty to pay for the improvement in accuracy that is apparently achievable by the use of the fourth-order scheme.

5. A NUMERICAL EXPERIMENT

To make the potential significance of the use of higher order schemes more evident, we performed a numerical experiment. The comprehensive study of Grammeltvedt (1969) was based on the numerical integration of an idealized barotropic flow in a cyclic channel. Polger (1971) performed a number of experiments with Grammeltvedt's equations, and we decided to test the higher order scheme with that computer program. The Initial Condition II used by Grammeltvedt was adopted. Three 72-hr forecasts

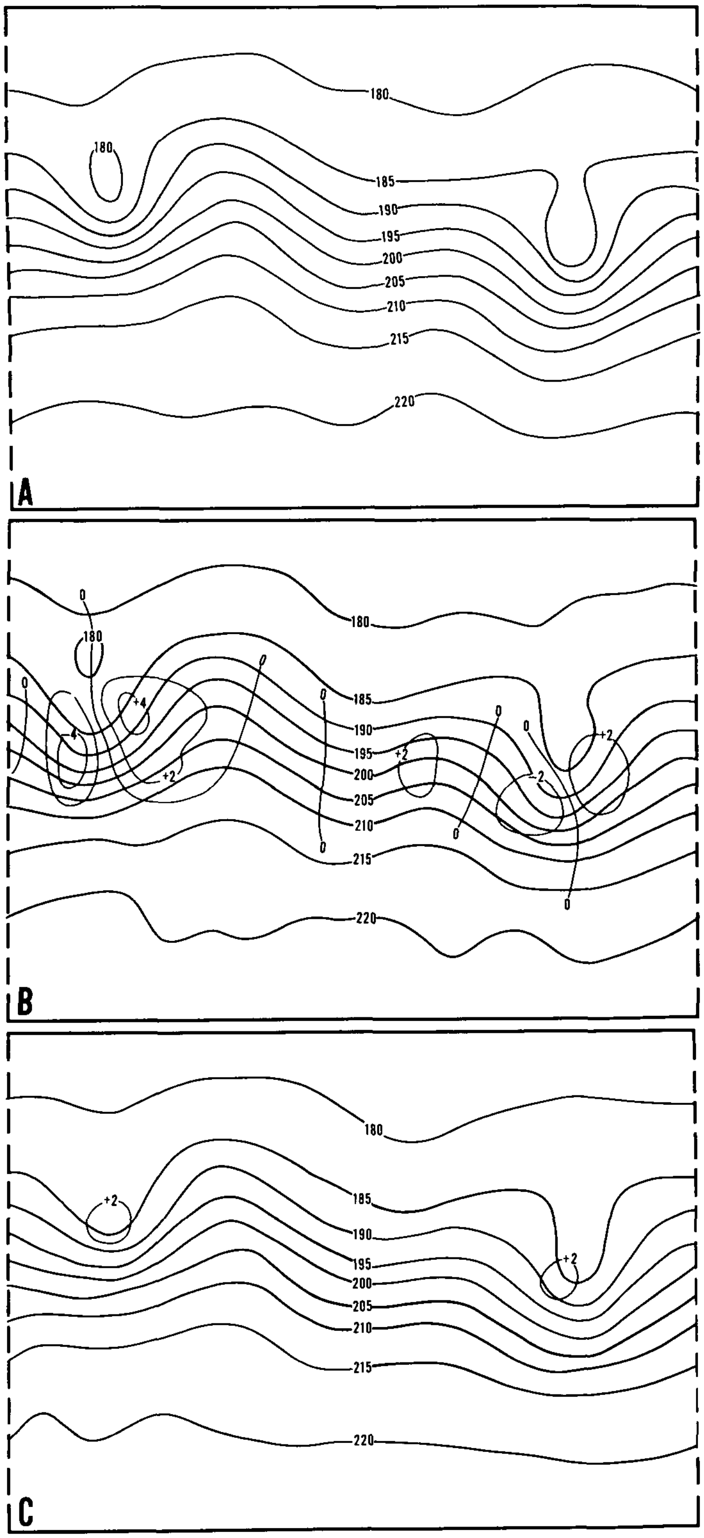

FIGURE 2.—The 24-hr forecasts of geopotential height by the (A) fine-mesh, (B) second-order, and (C) fourth-order schemes. The light lines in B and C show the difference field (relevant forecast less the fine-mesh forecast) isoplethed at intervals of 20 m.

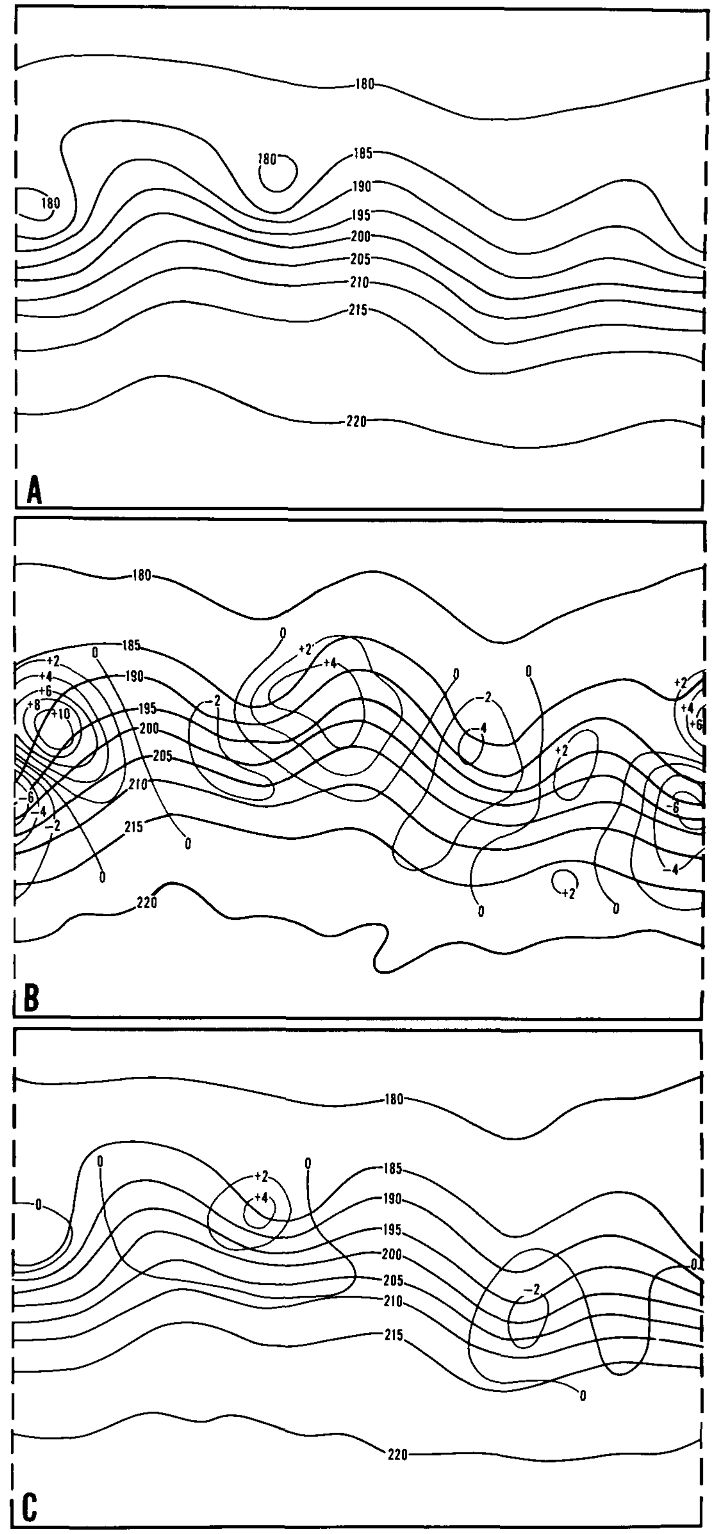

FIGURE 3.—Same as figure 2 for a 48-hr forecast.

were calculated. The second-order system,

$$\overline{u}^t_t+\overline{\overline{u}^{xy}\overline{u}^y_x+\overline{v}^{xy}\overline{u}^x_y+\overline{\phi}^y_x-\overline{f}^{xy}\overline{v}^{xy}}^{xy}=0,$$

$$\overline{v}^t_t+\overline{\overline{u}^{xy}\overline{v}^y_x+\overline{v}^{xy}\overline{v}^x_y+\overline{\phi}^x_y+\overline{f}^{xy}\overline{u}^{xy}}^{xy}=0,$$

and

$$\overline{\phi}^t_t+\overline{(\overline{u}^y\overline{\phi}^y)_x+(\overline{v}^x\overline{\phi}^x)_y}^{xy}=0,$$

was used on both a regular mesh and on a fine (one-half)

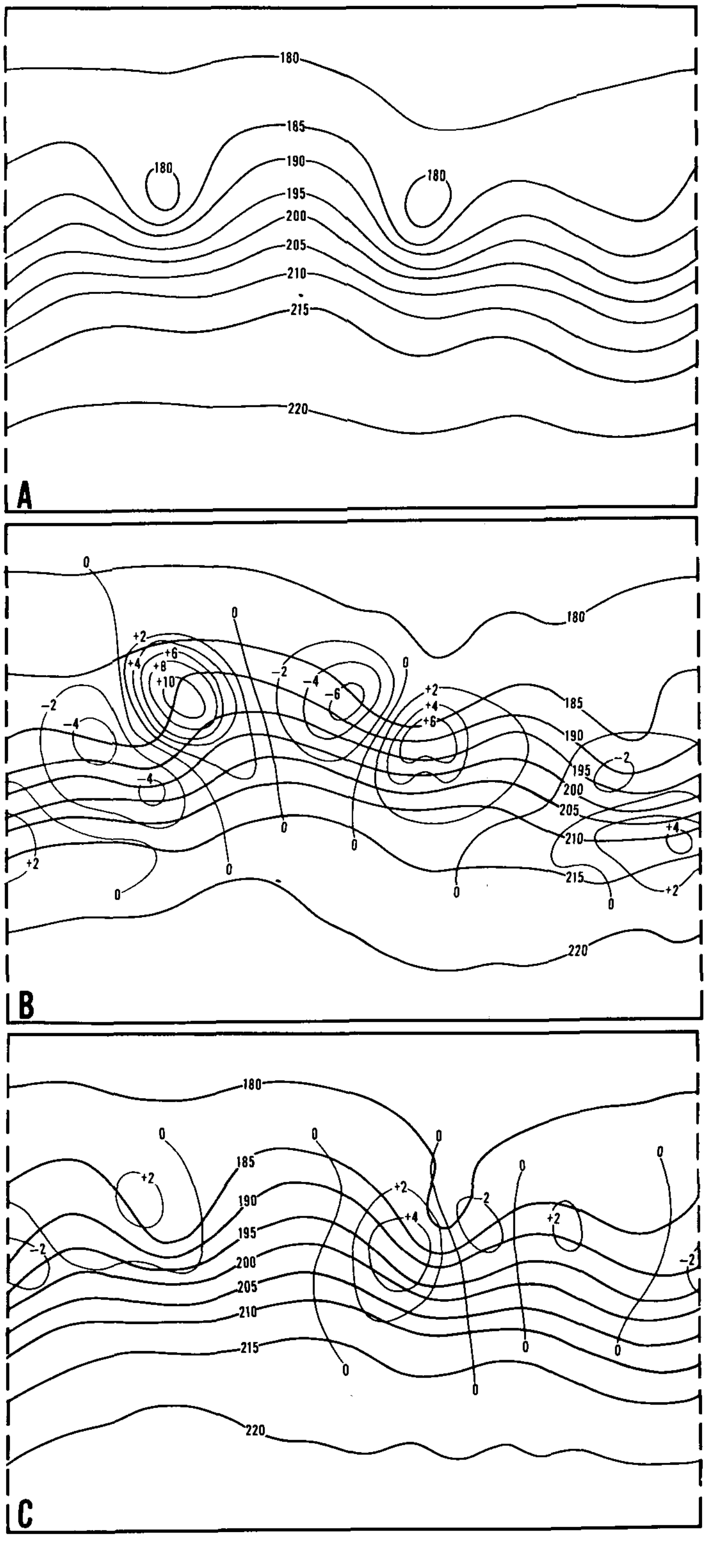

FIGURE 4.—Same as figure 2 for a 72-hr forecast.

mesh. The fourth-order scheme,

$$\overline{u}^t_t+\overline{\overline{u}^{xy}\overline{u}^{y_h}_{x_h}+\overline{v}^{xy}\overline{u}^{x_h}_{y_h}+\overline{\phi}^{y_h}_{x_h}-\overline{f}^{xy}\overline{v}^{xy}}^{x_h y_h}=0,$$

$$\overline{v}^t_t+\overline{\overline{u}^{xy}\overline{v}^{y_h}_{x_h}+\overline{v}^{xy}\overline{v}^{x_h}_{y_h}+\overline{\phi}^{x_h}_{y_h}+\overline{f}^{xy}\overline{u}^{xy}}^{x_h y_h}=0,$$

and

$$\overline{\phi}^t_t+\overline{(\overline{u}^{y_h}\overline{\phi}^{y_h})_{x_h}+(\overline{v}^{x_h}\overline{\phi}^{x_h})_{y_h}}^{x_h y_h}=0,$$

was solved only on the regular mesh.

We did not experiment with many alternate versions of the higher order scheme. It is fairly clear that alternative arrangements might have some advantages. The advecting winds were treated using second-order averaging to avoid smearing the wind maxima, the meridional profile of which is not well resolved on the regular grid.

Application of the fourth-order scheme near the y-coordinate boundaries was not possible. On the row once removed from the boundaries, the second-order scheme was used instead. The results of this experiment are depicted in figures 1–4. Figure 1 shows the initial contours of geopotential height. The first and third zonal harmonics are evident. Figures 2–4 are the fields at 24, 48, and 72 hr, respectively, as predicted by the fine mesh, regular mesh, and fourth-order schemes. The deviation of the regular mesh and fourth-order results from the fine mesh calculation are shown.

It is clear that the regular mesh solution departs quite considerably from the results obtained with the other schemes. The error field is rather characteristic of those found in operational forecasts. The smaller scale wave is translated too slowly.

From the theoretical estimate of truncation error, the fourth-order scheme should be somewhat more accurate than the fine mesh for the harmonics included in the initial data. There is some evidence that this is the case in the numerical experiment. The fourth-order scheme has translated the systems of troughs and ridges somewhat faster than did the fine-mesh calculation.

6. CONCLUSIONS

The results obtained, although quite limited, are of much more than academic interest. As noted by Wellck et al. (1971), the use of finer mesh sizes is extremely costly in computing requirements. While the methods used here were based on a difference scheme in advective form, Crowley's work (1968) indicates that the same techniques may be used with the conservative schemes that are preferred by many in general circulation simulation.

It should also be noted that higher order schemes are not an adequate substitute for reduced mesh sizes at the present time. We believe that a mesh size that adequately depicts the significant meteorological scales should be used. Once this has been accomplished, further grid reduction may be avoided by the implementation of more accurate difference schemes.

REFERENCES

Crowley, W. P., "Numerical Advection Experiments," *Monthly Weather Review*, Vol. 96, No. 1, Jan. **1968**, pp. 1–11.

Grammeltvedt, Arne, "A Study of Finite-Difference Schemes for the Primitive Equations for a Barotropic Fluid," *Monthly Weather Review*, Vol. 97, No. 5, May **1969**, pp. 384–404.

Lilly, Douglas K., "On the Computational Stability of Numerical Solutions of Time-Dependent Non-Linear Geophysical Fluid Dynamics Problems," *Monthly Weather Review*, Vol. 93, No. 1, Jan. **1965**, pp. 11–26.

Manabe, Syukuro, Smagorinsky, Joseph, Holloway, J. Leith, Jr., and Stone, Hugh M., "Simulated Climatology of a General Circulation Model With a Hydrologic Cycle: Effects of Increased Horizontal Computational Resolution," *Monthly Weather Review*, Vol. 98, No. 3, Mar. **1970**, pp. 175–212.

Miyakoda, K., "The Methods of Numerical Time Integration of One-Dimensional Linear Equations and Their Inherited Errors," *Journal of the Meteorological Society of Japan*, Ser. 2, Vol. 38, No. 6, Tokyo, Dec. **1960**, pp. 259–287.

Miyakoda, K., Strickler, R. F., Nappo, C. J., Baker, P. L., and Hembree, G. D., "The Effect of Horizontal Grid Resolution in an Atmospheric Circulation Model," *Journal of the Atmospheric Sciences*, Vol. 28, No. 4, May **1971**, pp. 481–499.

Platzman, George W., "The Lattice Structure of the Finite-Difference Primitive and Vorticity Equations," *Monthly Weather Review*, Vol. 86, No. 8, Aug. **1958**, pp. 285–292.

Platzman, Georrge W., "The Dynamical Prediction of Wind Tide on Lake Eie," *Meteorological Monographs*, Vol. 4, No. 26,s American Meteorological Society, Boston, Mass., Sept. **1963**, 44 pp.

Polger, Paul D., "A Study of Non-Linear Computational Instability for a Two-Dimensional Model," *NOAA Technical Memorandum* NWS NMC-49, U.S. Department of Commerce, National Meteorological Center, Washington, D.C., Feb. **1971**, 22 pp.

Richardson, Lewis F., *Weather Prediction by Numerical Process*, Dover Publications, New York, N.Y., **1965**, 236 pp.

Robert, André J., Shuman, Frederick G., and Gerrity, Joseph P., Jr., "On Partial Difference Equations in Mathematical Physics," *Monthly Weather Review*, Vol. 98, No. 1, Jan. **1970**, pp. 1–6.

Shuman, Frederick G., "Numerical Experiments With the Primitive Equations," *Proceedings of the International Symposium on Numerical Weather Prediction, Tokyo, Japan, November 7–13, 1960*, Meteorological Society of Japan, Tokyo, Mar. **1962**, pp. 85–107.

Wellck, Robert E., Kasahara, Akira, Washington, Warren M., and De Santo, Gloria, "Effect of Horizontal Resolution in a Finite-Difference Model of the General Circulation," *Monthly Weather Review*, Vol. 99, No. 9, Sept. **1971**, pp. 673–683.

[Received December 13, 1971; revised February 29, 1972]